2025.M32

2025L020A1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2025

Applied Mathematics

Higher Level

Tuesday 24 June Afternoon 2:00 - 4:30

400 marks

Examination Number

Date of Birth

For example, 3rd February 2005 is entered as 03 02 05

Centre Stamp

Instructions

There are ten questions on this paper. Each question carries 50 marks.

Answer any **eight** questions.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. All of your work should be presented in the answer areas, or on the given graphs, networks or other diagrams. Anything that you write outside of these areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in their simplest form, where relevant.

Diagrams are generally not drawn to scale.

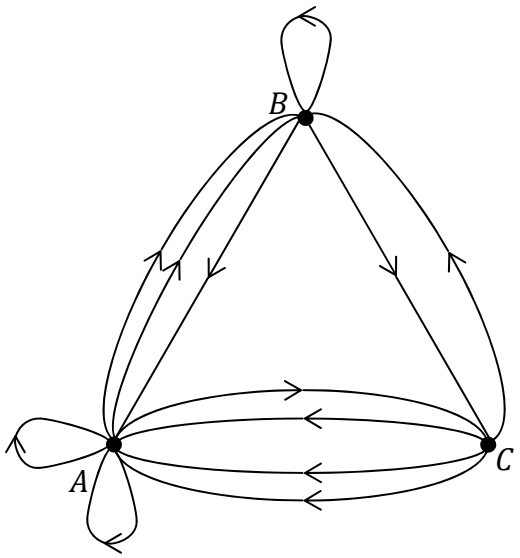
Unless otherwise indicated, take the value of g, the acceleration due to gravity, to be 9.8 m s^{-2}.

Unless otherwise indicated, $\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

Write the make and model of your calculator(s) here:

Question 1

(a) The diagram shows a directed graph.

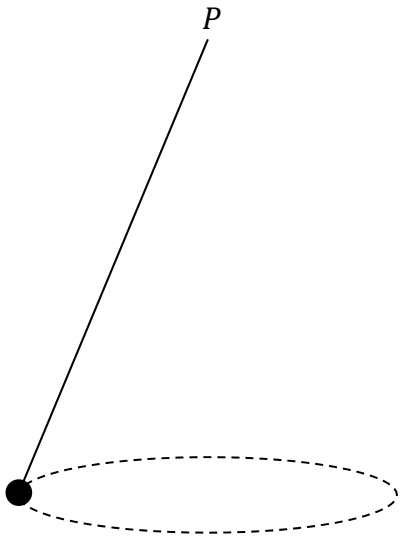


(i) Write the adjacency matrix, M, for this graph.

(ii) Calculate M^2.

(iii) What information is provided by the elements of M^2?

(b) A conical pendulum consists of a particle of mass 2 kg attached by a light inextensible string of length 1.3 m to a fixed point P. The particle moves in a horizontal circle of radius 0.5 m. The centre of the circle is vertically below P.



(i) Draw a diagram showing the forces acting on the particle.

(ii) Calculate the tension in the string.

(iii) Calculate the angular velocity of the particle.

(iv) The particle is given an increased velocity such that its period changes to 1.5 s.
Calculate the new radius.

Question 2

A particle of mass m moves vertically upwards through the air with displacement s and velocity v. Its motion may first be modelled by ignoring air resistance, so that it has constant acceleration a. At time $t = 0$ the particle has velocity $v_0 = 4$ m s^{-1} and displacement $s_0 = 0$.

(i) Using the chain rule, show that $a = v\frac{dv}{ds}$.

(ii) Use calculus to derive an expression for v in terms of a and s.

The model may be improved by including the force due to air resistance as $\frac{1}{40}mv^2$.

(iii) Show that the upward motion can now be expressed by the differential equation:

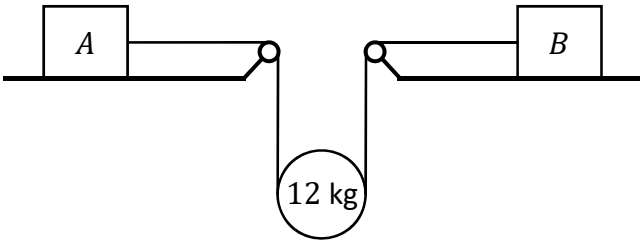
$$\frac{2v}{v^2 + 392}dv = -\frac{1}{20}ds$$

(iv) Solve this differential equation to find an expression for v in terms of s.

(v) Calculate the greatest height the particle will reach.

(vi) By using $a = \frac{dv}{dt}$ solve a differential equation to find an expression that relates v and t.

Question 3



(a) Block A of mass 8 kg and block B of mass 4 kg lie at rest on two rough horizontal tables.

The coefficient of friction between A and the table it lies on is μ_A.
The coefficient of friction between B and the table it lies on is μ_B.

$\mu_B = 2\mu_A$

The blocks are connected by a light inextensible string which passes under a smooth movable pulley of mass 12 kg.

After the system is released from rest the blocks and the pulley move and the string has a tension of 32 N.

(i) Calculate μ_B.

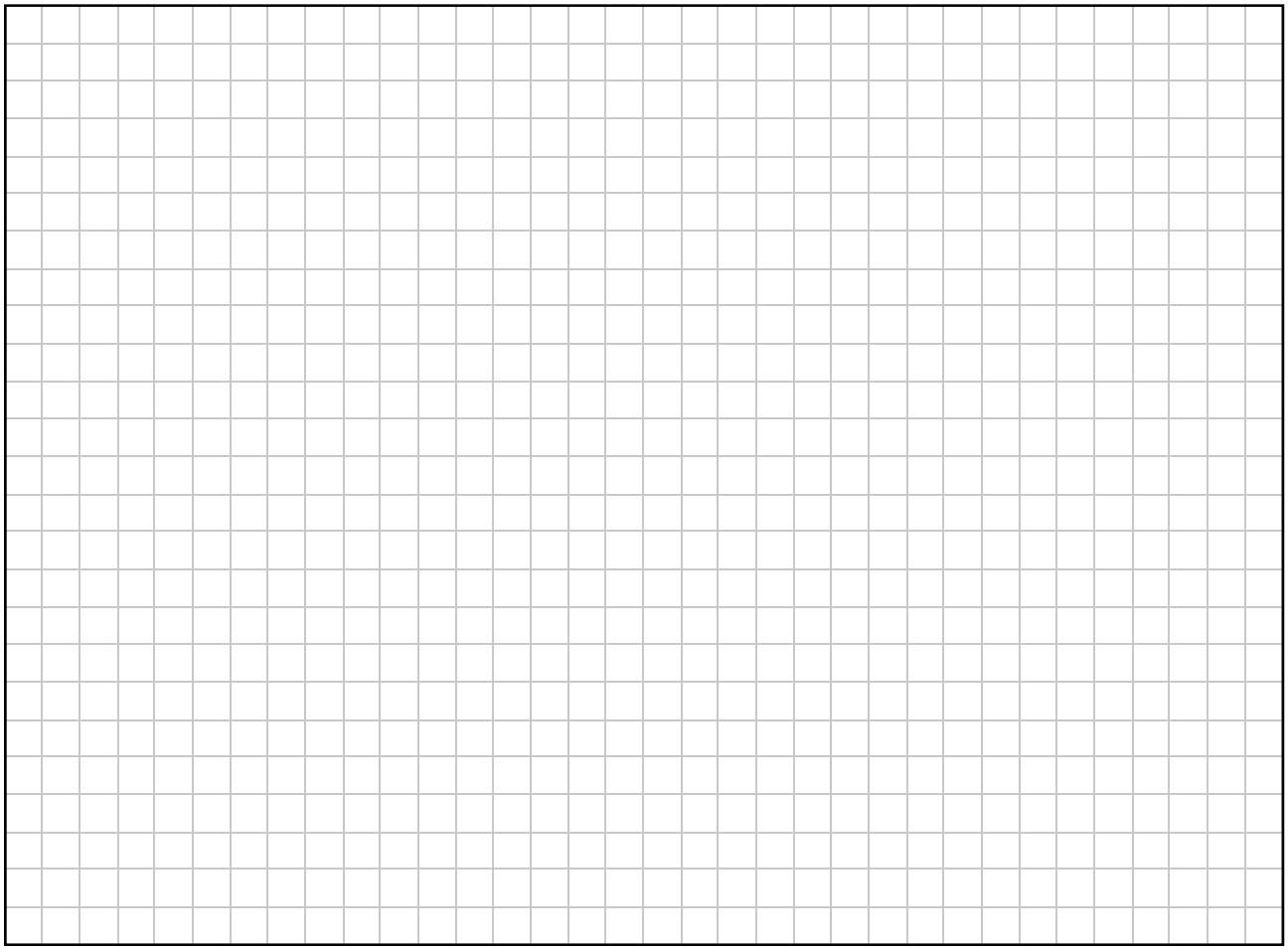
(ii) Calculate the acceleration of the movable pulley.

(b) An electrician needs to link 7 landline telephones in an office. The telephones, A to G, need to be connected by cables within the same electrical network.

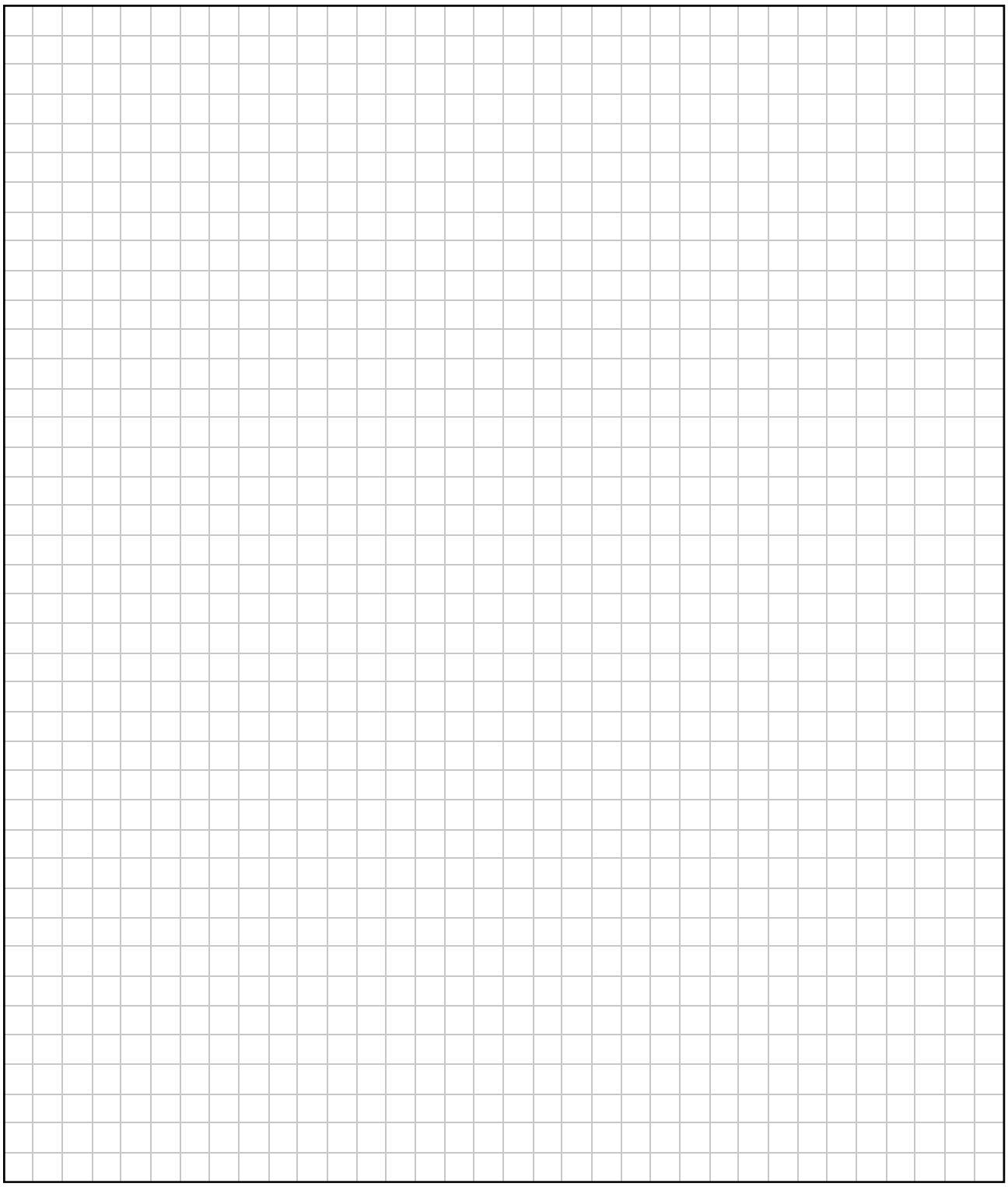
The table below shows the distance, in metres, between the different telephones.

	A	B	C	D	E	F	G
A	–	9	14	13	12	15	18
B	9	–	13	14	12	9	15
C	14	13	–	11	13	14	16
D	13	14	11	–	10	9	14
E	12	12	13	10	–	11	12
F	15	9	14	9	11	–	8
G	18	15	16	14	12	8	–

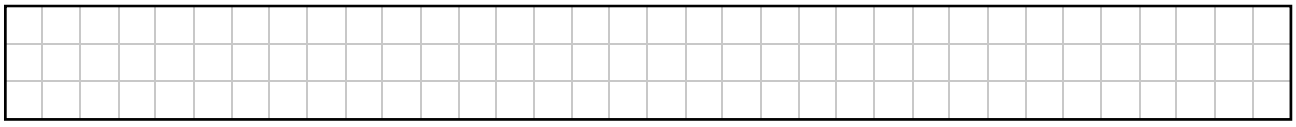
(i) The electrician wishes to use the minimum length of cable required to connect every telephone within the same electrical network. Starting at telephone A, use Prim's algorithm to calculate the minimum length of cable required.



(ii) Draw the electrical network that uses the minimum length of cable.



(iii) Name another algorithm that could be used to calculate the minimum length of cable required.

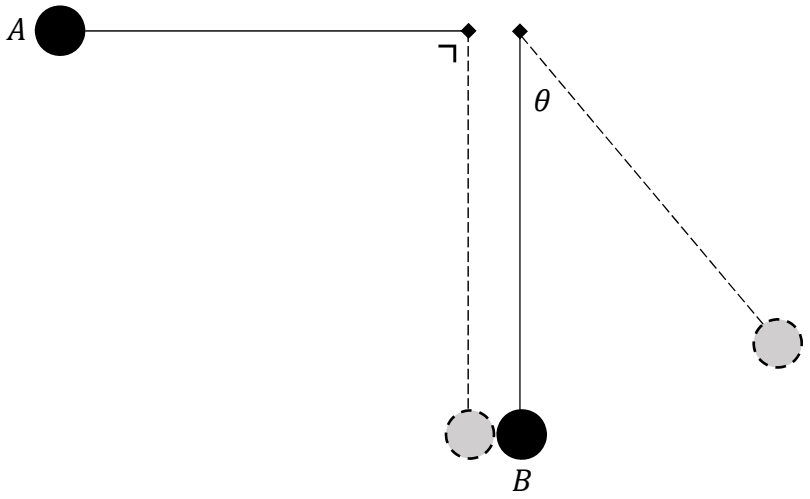


Question 4

Two small smooth spheres, A of mass 2 kg and B of mass 1 kg, are attached to pegs of equal height by two light inextensible strings, each of length 30 cm.

The spheres are initially at rest. A is released when the string is taut and horizontal.

A collides with B, which is suspended vertically.



The coefficient of restitution between the spheres is $\frac{3}{50}$.

(i) Show that A strikes B with a speed of $\frac{7\sqrt{3}}{5}$ m s^{-1}.

(ii) Calculate the speed of each sphere immediately after the collision.

(iii) Show in a diagram the forces acting on B when the string makes an angle θ with the downward vertical.

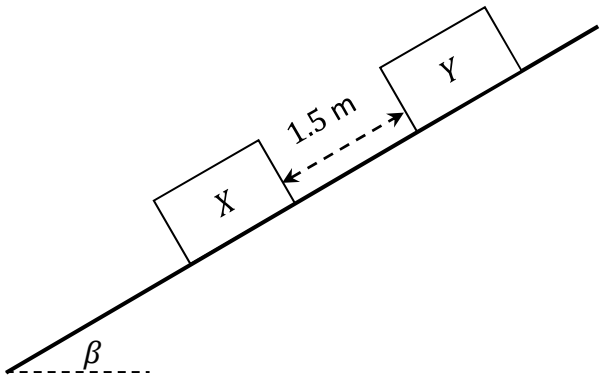
(iv) Write an expression, in terms of θ, for the height B has moved through when the string makes an angle θ with the downward vertical.

(v) Write an expression, in terms of θ, for the speed of B when the string makes an angle θ with the downward vertical.

(vi) Calculate the value of θ such that the string attaching B has a tension of 15 N.

Question 5

(a) Block X and block Y are held separately at rest on a rough plane inclined at β to the horizontal. X and Y are 1.5 m apart, as shown.



The coefficient of friction between X and the plane is $\frac{1}{2}$ and the coefficient of friction between Y and the plane is $\frac{2}{5}$. The blocks are released from rest.

(i) Show, on separate diagrams, the forces acting on the blocks while they are moving.

(ii) Calculate the acceleration of each block in terms of β.

The blocks collide 2 s after they are released.

(iii) Calculate β.

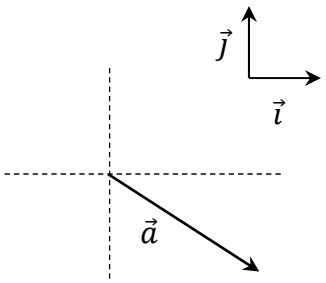
(b) $\vec{a} = 14\vec{\imath} - 9\vec{\jmath}$

$\vec{b} = p\vec{\imath} + q\vec{\jmath}$

$|\vec{b}| = 21$

$\vec{a} \perp \vec{b}$

Calculate the possible values of p and q.



Question 6

(a) Éanna wishes to purchase a new campervan. He wants to have the use of a campervan for five years and to minimise how much money this will cost him.

He does not wish to own a campervan at the end of the five years.

The cost of a new campervan is €80 000.

A campervan must be serviced each year.
The cost of servicing a campervan during its first year is €500.
The cost of servicing a campervan during its second year is €800.
The cost of servicing a campervan during its third year is €1400.

A one year old campervan has a resale value of €74 000.
A two year old campervan has a resale value of €70 000.
A three year old campervan has a resale value of €61 500.

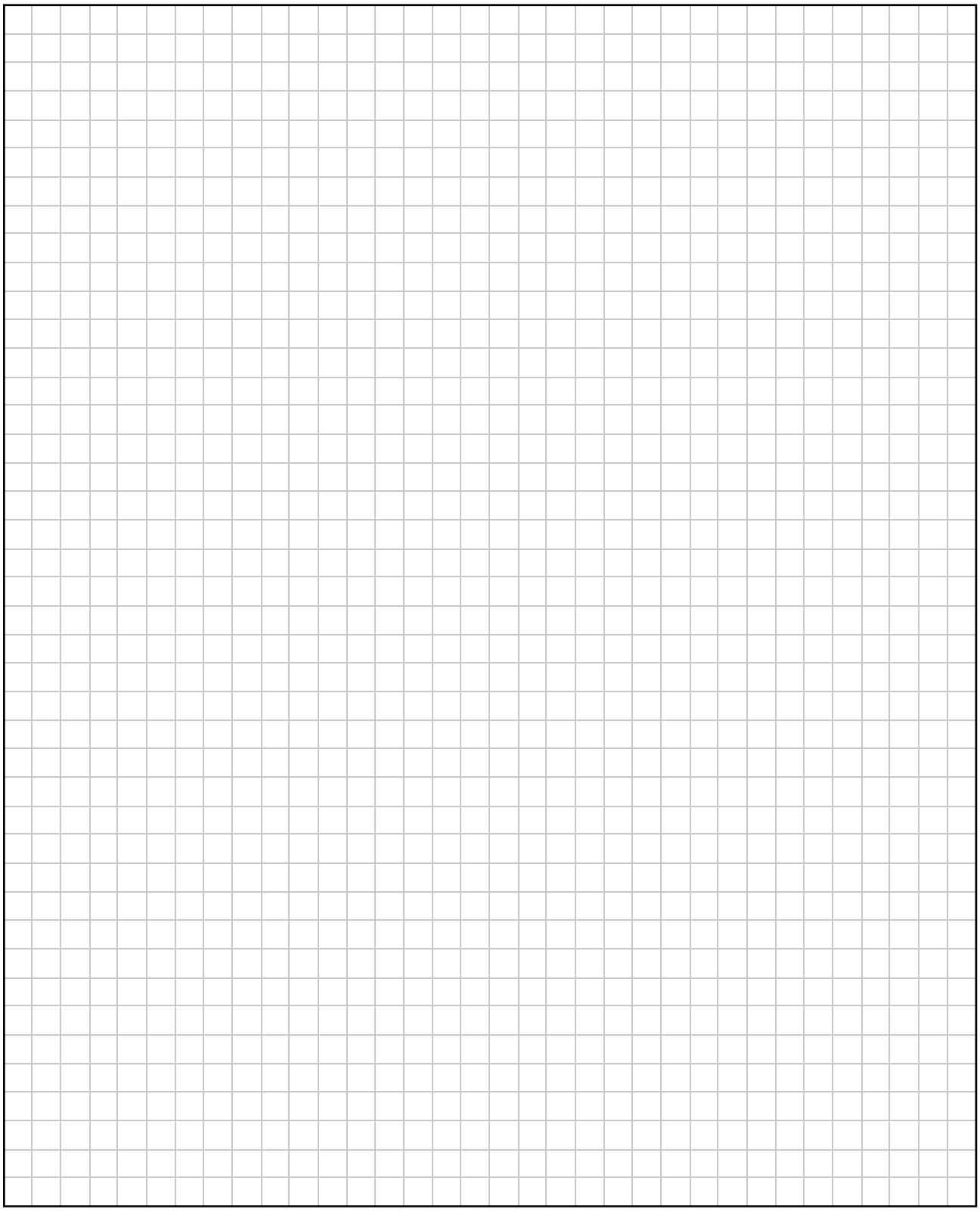
Éanna wishes to own a campervan that is no more than three years old at all times during the five year period. Any time he purchases a campervan, it is a new one.

(i) Use Dynamic Programming to find Éanna's optimal strategy. Calculate how much it will cost Éanna if he uses this optimal strategy. Relevant supporting work must be shown.

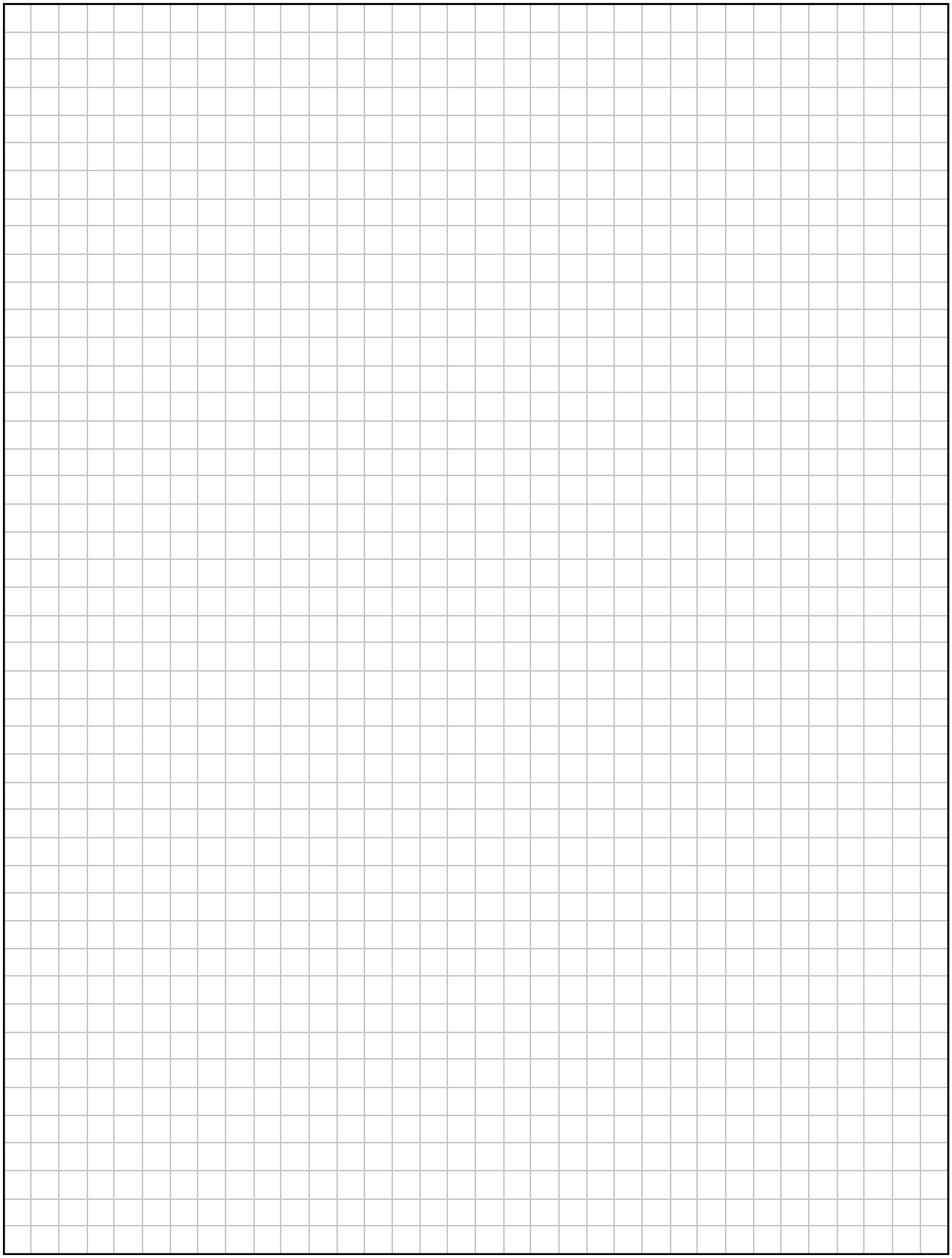
(ii) In the context of this question, distinguish between the concepts of *stage* and *state*.

(b) Two students, Áine and Brody, are investigating the properties of an elastic resistance band in their school gym. The band has a natural length of 1 m and an elastic constant of 650 N m^{-1}. One end of the band is attached to a fixed pole.

(i) Use integration to calculate the work done by Áine in extending the band horizontally to a length of 1.2 m.

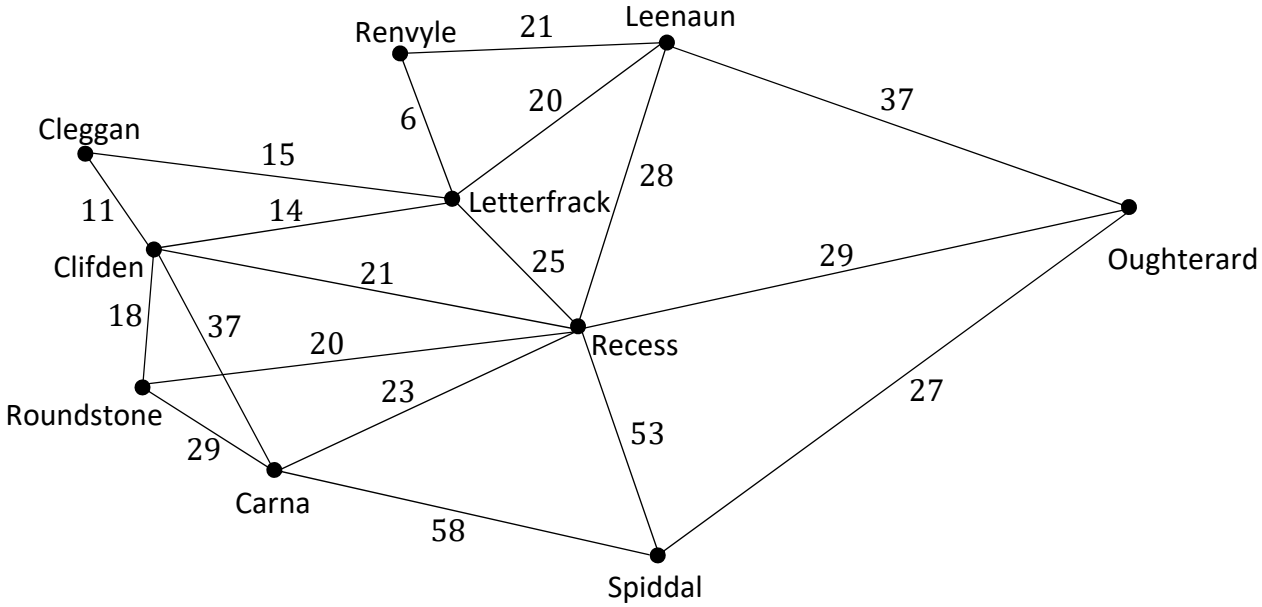


(ii) Brody takes the extended band and extends it further. He claims to have done twice the work that Áine did. If Brody is correct, calculate the new length of the band.



Question 7

(a) Shauna wishes to tour Connemara. While planning her route, she uses a road map to draw the network shown below. The weight of each edge represents the distance (in km) between each location.

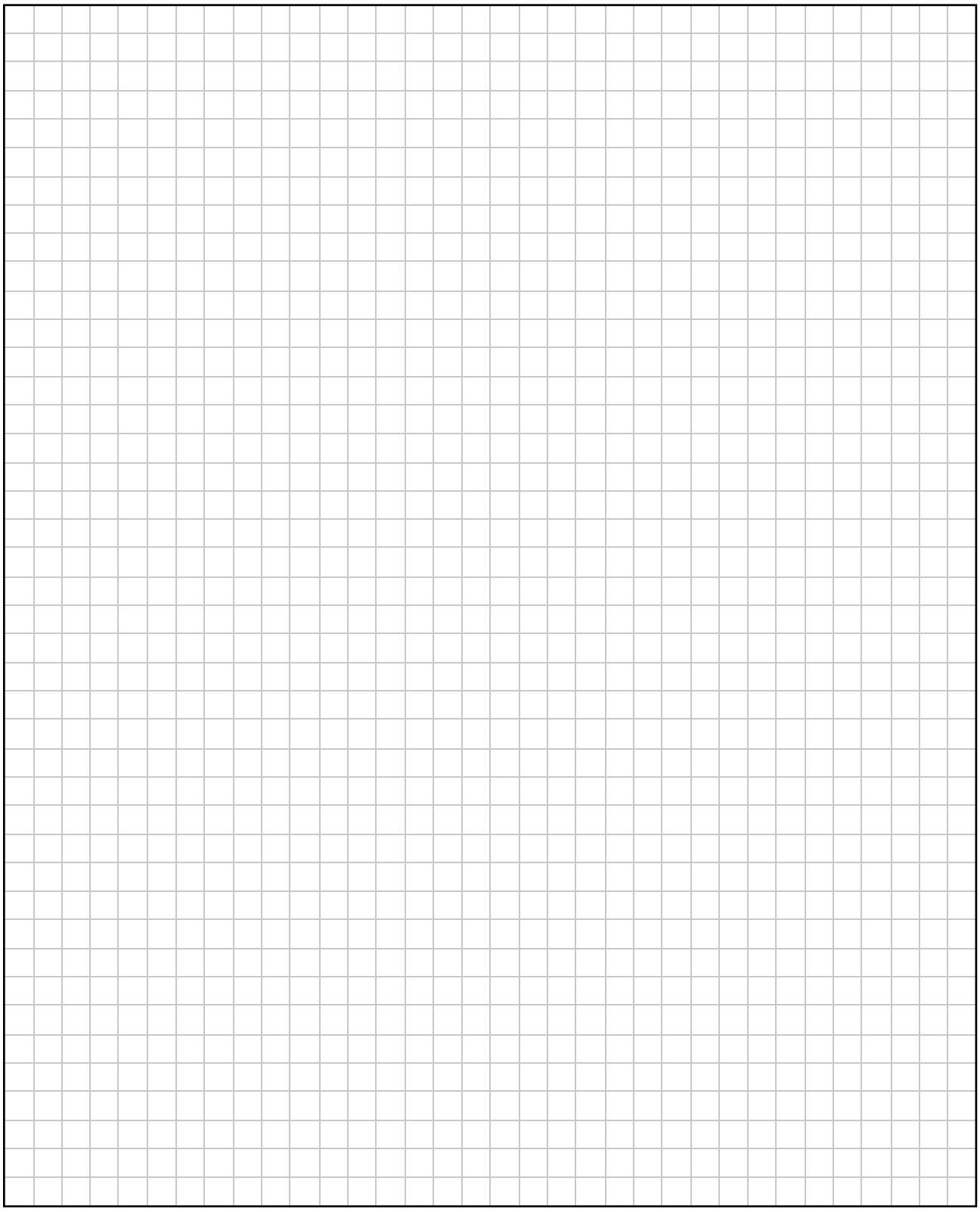


(i) Use Dijkstra's algorithm to find the shortest path from Oughterard to Cleggan. Calculate the length of the shortest path. Relevant supporting work must be shown.

(ii) Describe how a network differs from a road map.

(b) Chioma is throwing a basketball against a wall. The ball leaves her hands at chest height, 1.35 m above the ground. The ball hits the wall at a height of 2 m. Chioma is standing 3 m from the wall when she throws the basketball with an initial speed of 6.3 m s^{-1} at an angle α to the horizontal.

(i) Calculate the two possible values of α.



(ii) For the smaller value of α, calculate the velocity of the basketball as it hits the wall.

The coefficient of restitution between the basketball and the wall is $\frac{3}{7}$.

(iii) For the smaller value of α, calculate the horizontal distance between the wall and where the basketball lands on the ground for the first time.

Question 8

(a) On Monday morning, the symptoms of a specific infection were detected in 5 students. On Tuesday morning, the symptoms were detected in an additional 12 students.

A local doctor predicts that the increase in the number of new infections detected on a given day will be twice the increase in the number of new infections detected on the previous day.

The doctor wishes to model this prediction as a difference equation so as to determine U_n, the number of new infections detected on day n.

(i) Write down a difference equation to express U_{n+2} in terms of U_{n+1} and U_n, where $n \geq 0$, $n \in \mathbb{Z}$.

(ii) Is this a homogeneous difference equation or an inhomogeneous difference equation? Explain your answer.

(iii) Solve the difference equation to find an expression for U_n in terms of n.

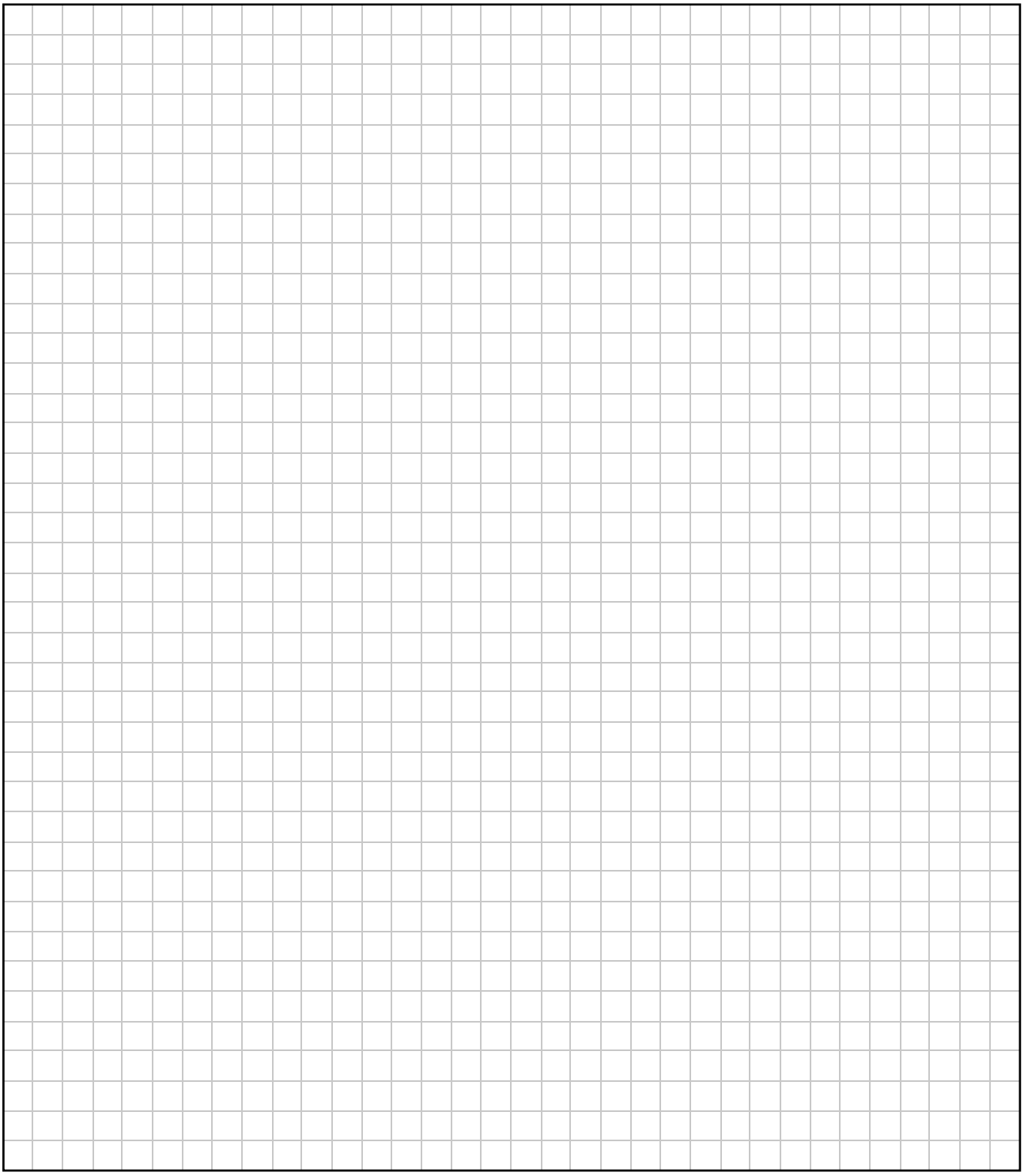
(iv) Calculate the total number of infections detected between Monday morning and Friday evening.

(b) The population growth in a certain region can be modelled by the following differential equation:

$$\frac{dP}{dt} = t^2 e^t$$

where P is the population and t is the time measured in years.

(i) Using integration by parts or otherwise, derive an expression for $P(t)$, the population at any time t, given that $P(0) = 2200$.

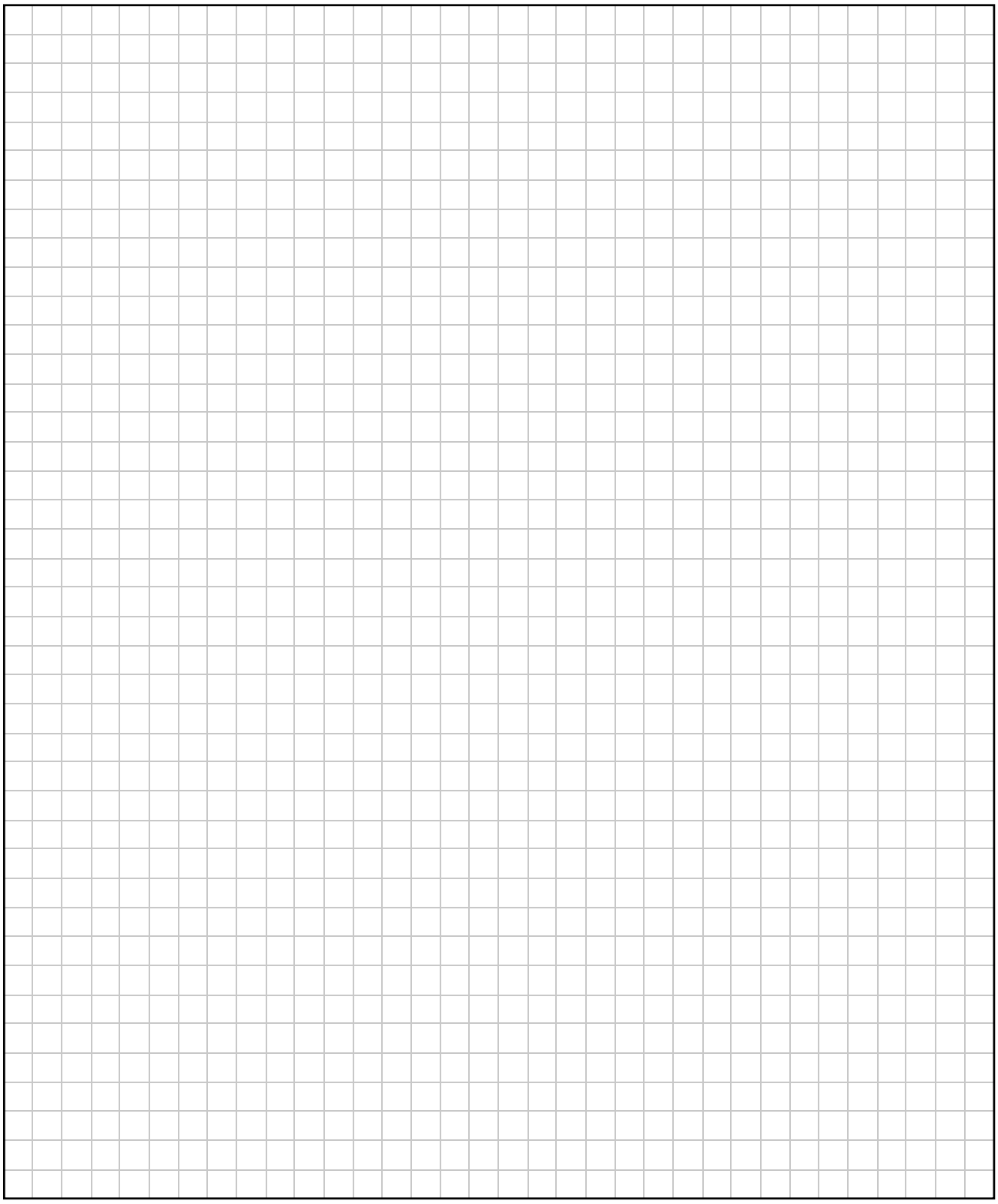


(ii) Calculate $P(5)$, the population after 5 years.

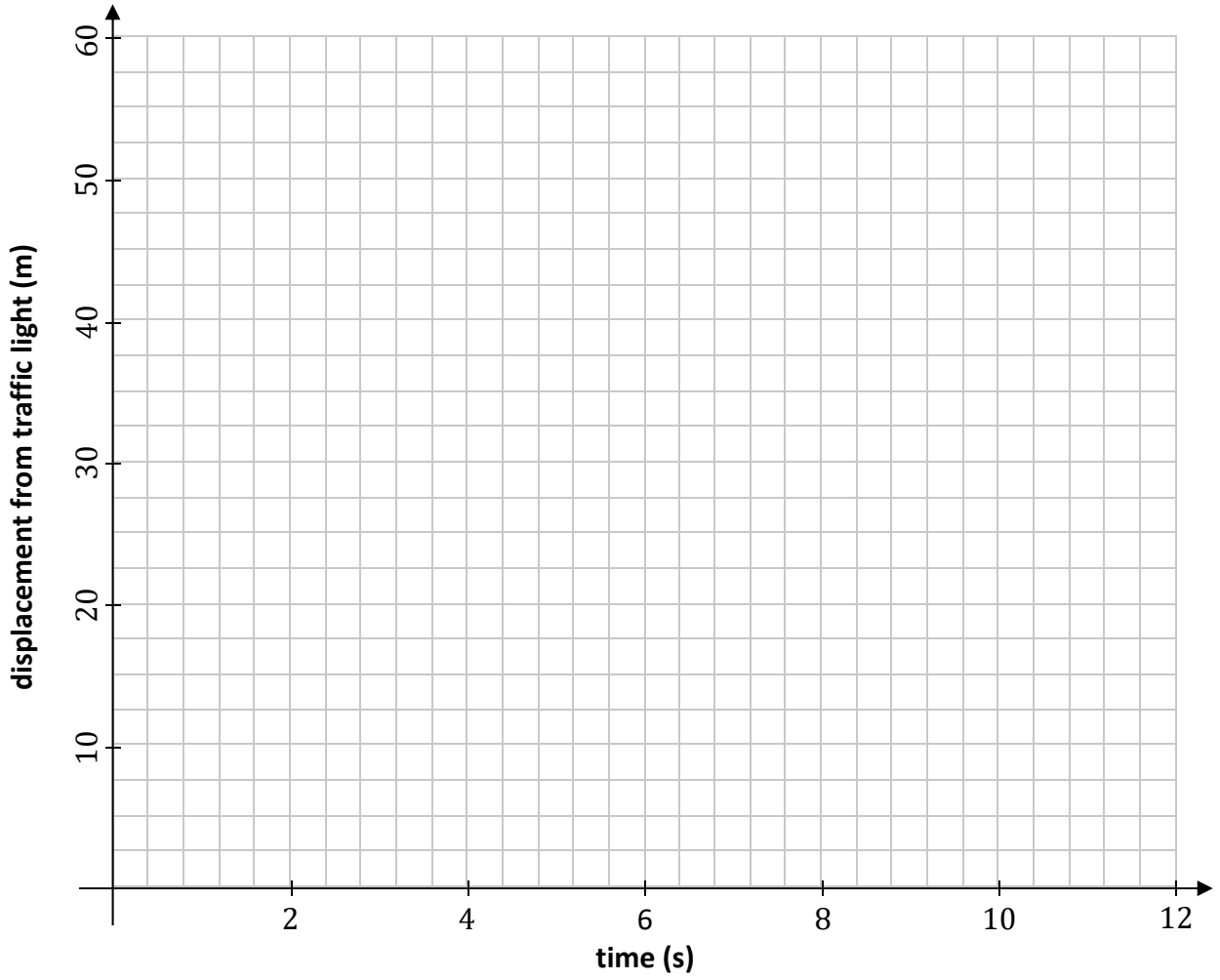
Question 9

(a) A car, using cruise control, is travelling with a constant velocity of 50 km hr^{-1}.
The driver sees a traffic light 60 m ahead change from green to orange.
The driver takes half a second to react to this.

(i) Calculate the constant deceleration required to stop the car at the light.



(ii) Using the axes below, draw a displacement-time graph for the motion of the car from when the driver sees the traffic light change. Relevant calculations should be shown in the space provided.



(b) Two small smooth spheres A and B, of masses $3m$ and m respectively, collide obliquely. The velocity of A before the collision is $3\vec{\imath} + 4\vec{\jmath}$ and the velocity of B before the collision is $-2\vec{\imath} + \vec{\jmath}$, where the $\vec{\imath}$ axis is along the line joining the centres of the spheres at the point of impact.

The coefficient of restitution is $\frac{1}{3}$.

Show that the fraction of kinetic energy lost as a result of the collision is $\frac{5}{24}$.

Question 10

(a) A new streaming media service is trying to increase its total number of customers.
The company introduces a strategy to attract new customers.

The company develops a difference equation to predict S_n, the total number of customers in month n. The difference equation is:

$$S_{n+2} = \frac{1}{4}(S_{n+1} + 5S_n) - 15n + 40$$

where $n \geq 0$, $n \in \mathbb{Z}$.

Immediately before the introduction of the new strategy, the company has 2016 customers, i.e. $S_0 = 2016$. After one month the company has 3500 customers, i.e. $S_1 = 3500$.

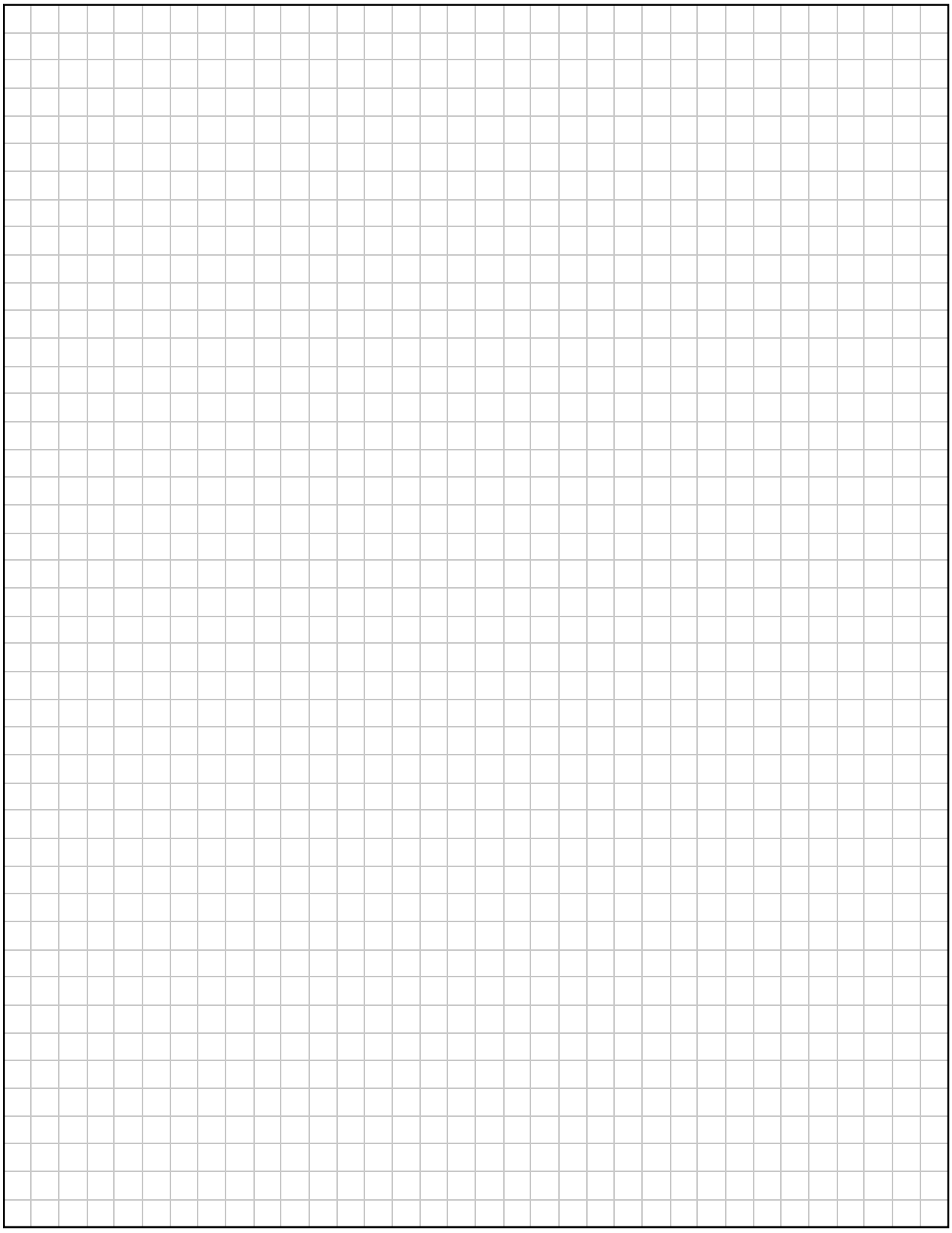
(i) Write down the values of S_2 and S_3.

(ii) Solve the difference equation to find an expression for S_n in terms of n.

(iii) The strategy is deemed successful if the service has at least $35\,000$ customers after one year. Is the strategy successful? Justify your answer.

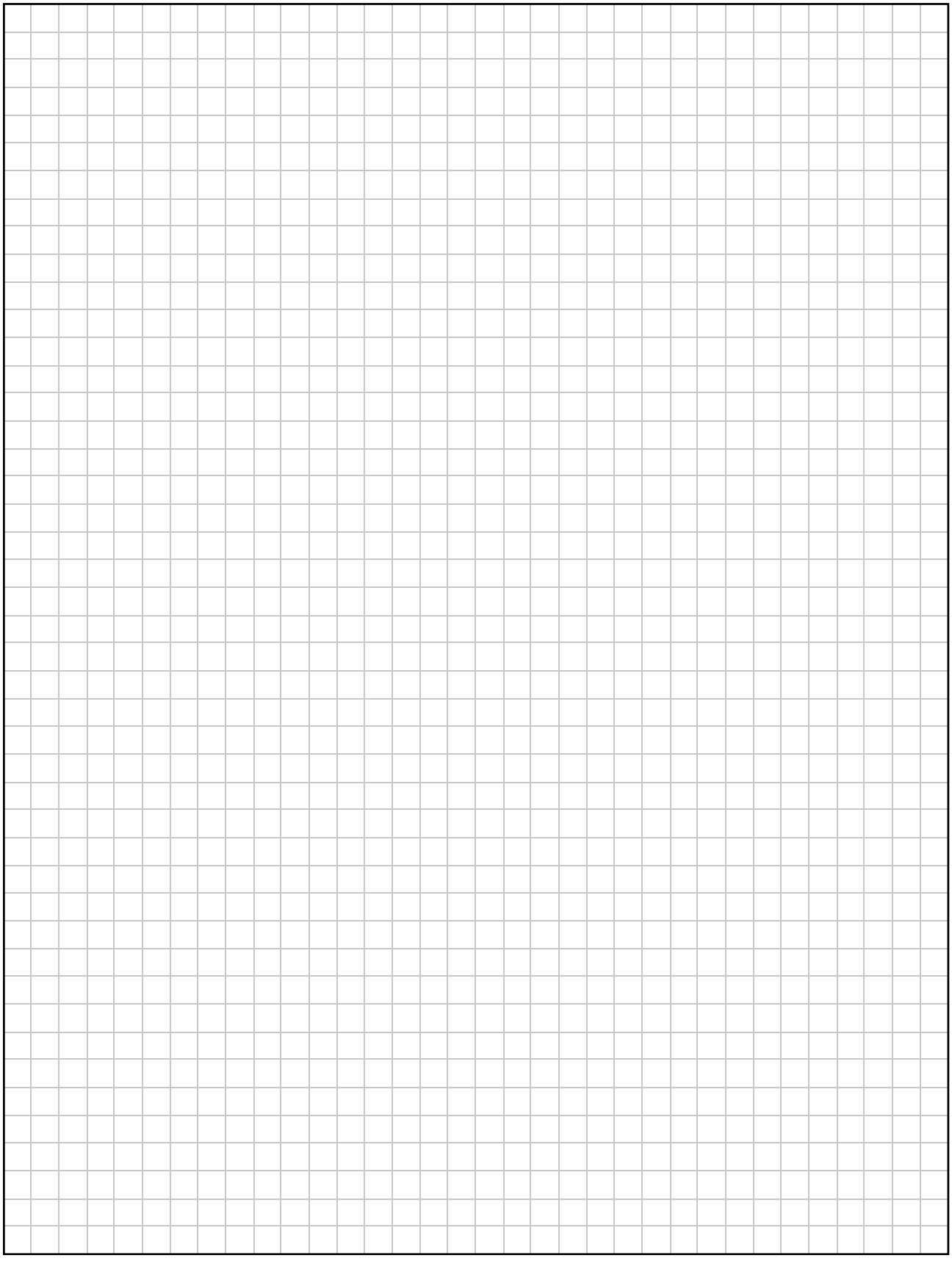
(b) A particle is projected from a point P with initial speed u. After 6 s its horizontal displacement and vertical displacement are 75 m and 30 m respectively.

Calculate u.



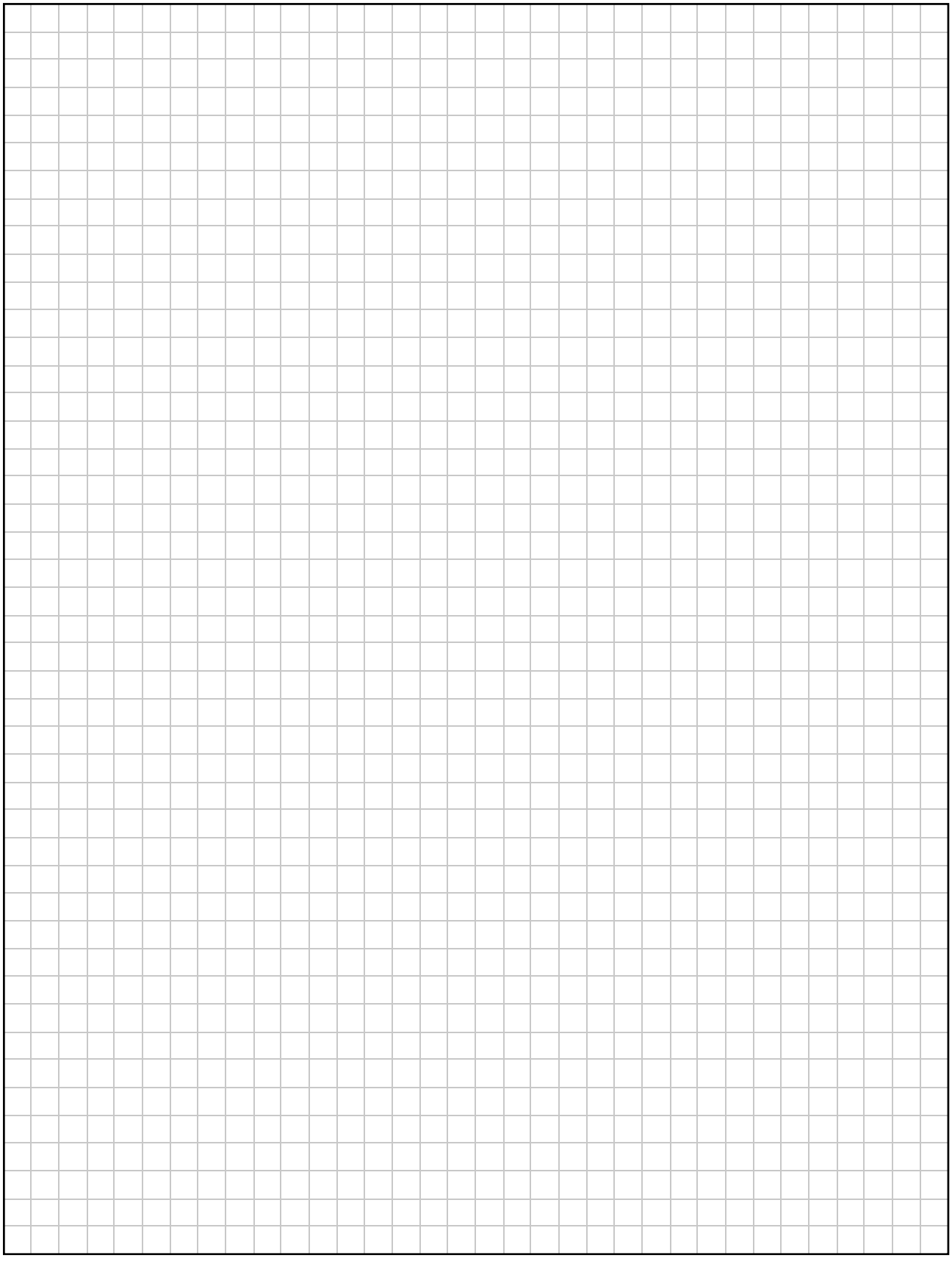
Page for extra work.

Label any extra work clearly with the question number and part.



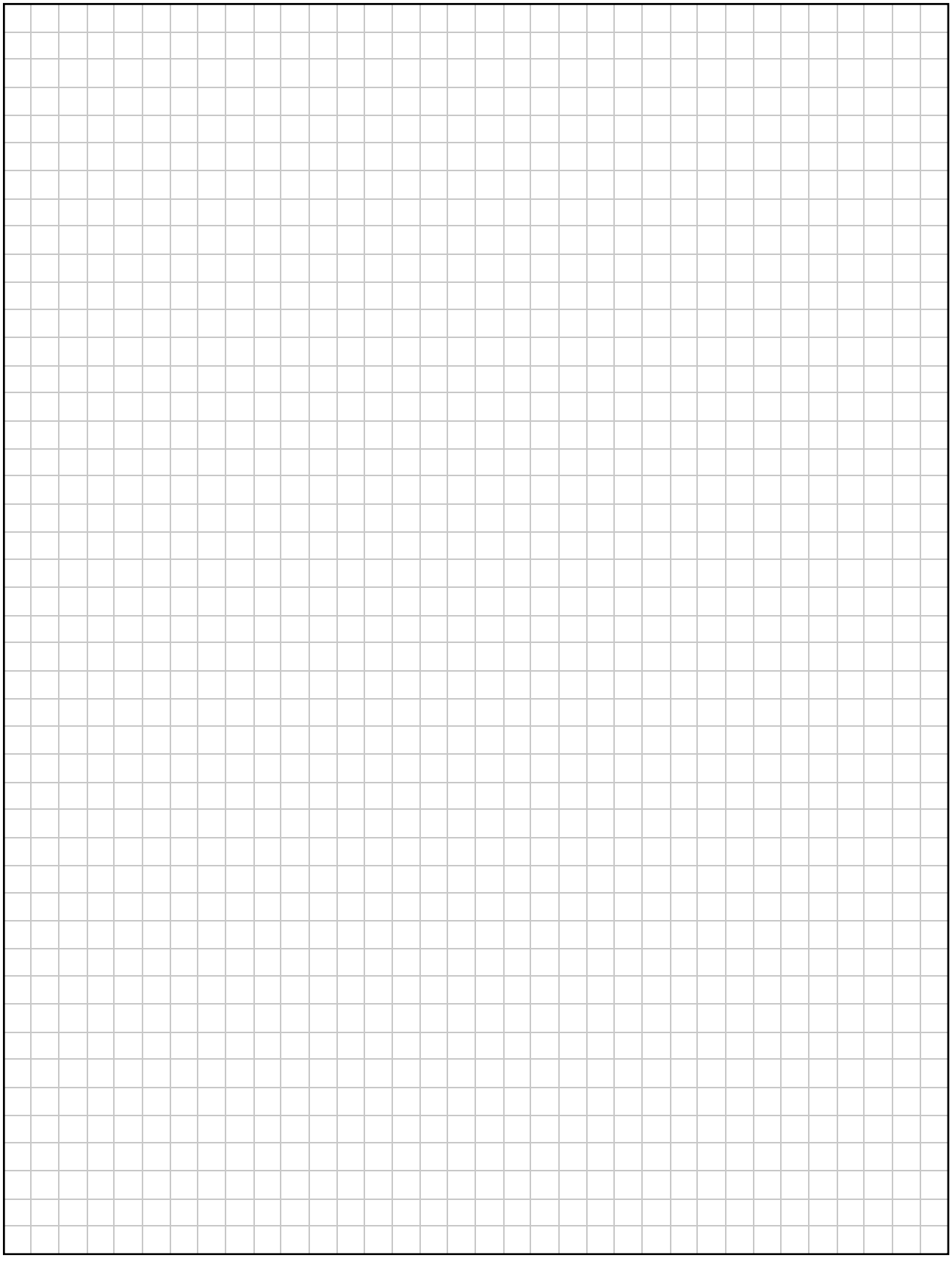
Page for extra work.

Label any extra work clearly with the question number and part.



Page for extra work.

Label any extra work clearly with the question number and part.



Do not write on this page

Leaving Certificate – Higher Level

Applied Mathematics

Tuesday 24 June

Afternoon 2:00 - 4:30